"Attaining mathematical competencies via the use of other subjects in a first year mathematics course at an agricultural university"

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1. Introduction

In their first term at the Royal Veterinary and Agricultural University (RVAU) in Copenhagen, Denmark, students in agriculture, forestry, food science, biotechnology and agricultural economics (about 250 students in total) do a basic mathematics course in calculus and linear algebra. The course has been reasonably popular for a number of years, but the students' high school background as well as the needs of the other subjects at RVAU has changed, and we gradually felt that the course was no longer up to date. Furthermore, we wished to let applications play a bigger role in the course. During the last two years we have revised the course, and this paper focuses on two aspects of the revision: firstly, the inspiration from the recently published Danish KOM report on mathematical competencies in the learning of mathematics, and, secondly, the influence which a series of meetings, held prior to the revision, with representatives from other subjects at RVAU had on it. We will also describe our use of examples from applications in the course.

2. Competencies

The "KOM report" (Niss et al., 2002) by a working group under the Danish Ministry of Education has among its aims to describe curricula in a way so that one can determine what it means to master mathematics, and thereby describe progression in the teaching and learning of mathematics. To this end it uses the notion of mathematical competencies (from now on referred to simply as competencies). Briefly, such a competence can be described as "a well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge" (Niss et al., 2002, p.43). The report defines eight central mathematical competencies divided in two groups. The group *being able to ask and answer in, with and about mathematics* consists of the *mathematical thinking, problem solving, modelling* and *reasoning* competencies, whereas the group *being able to deal with mathematical language and tools* consists of the *representation, symbol & formalism, communication* and *aids & tools* competencies.

The fairly well established curriculum of the course before the revision was developed and described in terms of topics. When we began the revision, we found that viewing the course from the perspective of competencies supplemented the topic-based perspective in a useful way. We assessed the importance in the course and used this as a basis on which to rethink the curriculum of the course and the way the material was presented. (A *normative* (Niss et al., 2002, p.71) use of the framework of competencies.)

3. Meetings with representatives from other subjects

Since mathematics is a service subject at RVAU, the main justification of the course lies in the use of mathematics in other courses at RVAU. Because of this, we decided to start the revision with a series of meetings with representatives from other subjects with the aim of getting as broad an understanding as possible of what the other subjects need mathematics for. It is our clear feeling that the meetings to a high extent served this purpose. In the following presentation we have divided the responses from the representatives into “topics” and “competencies”.

Topics

There were no major surprises among the topics requested by the representatives, but it was interesting to observe the frequency with which the various topics were mentioned. Among the most frequently mentioned were properties and differentiation of basic functions (logarithmic, linear, power and exponential functions) as well as investigation of functions with parameters. (In Danish high schools, investigation of functions (curve sketching) is nowadays to a high extent done with the help of graphical calculators and with less emphasis on the “classical” approach involving the derivative. Consequently the students are not sufficiently prepared to handle functions involving parameters.)

On the other hand, we were slightly surprised by the fact that some topics included in the course were not considered important by any of the representatives. This went for instance for planar integrals and general first order differential equations (whereas differential equations for exponential and logistic growth were considered important).

Competencies

Apart from concrete topics, a number of the representatives also mentioned more vaguely defined abilities needed in their courses. In some cases we prompted them with phrases such as “general mathematical abilities”, “mathematical notions” and “mathematical thinking”, whereas we in general tried to avoid the word “competencies”, since this word has become very fashionable and has acquired a range of different meanings. We found many of the responses relating to these abilities very interesting and will therefore discuss them in more detail than we did with the topics.

To be able to handle equations

This was phrased in many different ways, for instance “to solve equations”, “to work with an equation; insert things and rearrange a bit” or “to manipulate with simple functions”. A typical situation could be to isolate a quantity (variable or parameter) from an equation. Here the ability includes an understanding of what has to be done and why as well as the technical skills involved (taking the logarithm to “get rid of” an exponential etc.).

To be able to model a given situation using mathematics

When we asked in more detail about modelling, it appeared that it is only later on in their lines of study that the students need to be able to actively model a situation. In their first couple of years it is sufficient that they can work with the models presented to them. In particular, the ability to handle and interpret parameters was considered very important. Another frequently stated need was for the students to be able to “jump back and forth between mathematics and other subjects”.

Not to be afraid of mathematics

As an example of this, some of the representatives mentioned that in the jumping back and forth between mathematics and other subjects (see above), the students tended to lose the thread of an argument, whenever they “jumped into mathematics”. This was not necessarily because the mathematics was difficult, but the mere fact of its presence seemed to make some students panic. We would probably not choose to consider “absence of fear of mathematics” as a competence in itself, but rather think that the best way to handle the fear is frequent exposure to and positive experiences from an early age of situations where mathematics interplays with realistic problems.

To understand what is happening
This is of course a very broad statement, but the word “understanding” appeared frequently at the meetings. The main attitude among the representatives was that if the students have a good basic understanding of mathematics and are prepared to meet mathematics in other subjects, then the other subjects can present further mathematical topics themselves if necessary.

Examples from applications
Even before the revision, the course contained a number of examples from applications. However, since we were planning to make more active use of such examples, we needed to gather additional examples that were both realistic and which could help us attain our learning goals (see section 4 for more details). Many classical applications (e.g. from physics) are not particularly relevant at RVAU, and the meetings turned out to be good starting points for gathering useful examples: the representatives agreed on the importance of relevant examples and either directly provided us with examples or guided us to the relevant parts of their textbooks and other course material.

Additional benefits from the meetings
The other subjects were generally very positive about our initiative and had a welcoming and interested attitude to mathematics at RVAU. Through the meetings we made useful connections to other people at RVAU and in this way the meetings helped to further the integration of mathematics into the university community and improve the “standing” of mathematics at RVAU. We think that these “socialising” effects of the meetings should not be underestimated.

4. Decisions and conclusions about the new course

Our decisions about the new course were mainly based on

- the old course including our impression of its weaknesses,
- the inspiration from the KOM report,
- the responses from the representatives and the collected examples from applications.

During and after the first run of the new course we tried to observe how well our proposed changes and ideas were implemented and whether unexpected phenomena occurred. These observations were then used when we prepared the second run of the course (similar to the *didactical engineering* of Artigue (1994)). The following description is mainly of the second run.

Topics and examples from applications
Compared to the old version of the course, we intended to focus more on certain competencies (see below) and to spend more time on examples from applications. It was therefore necessary to consider whether certain areas of the course could be omitted or at least reduced. Already before the meetings with the other subjects there were some topics in the course (such as inverse trigonometric functions) that we did not find sufficiently relevant to retain. The meetings confirmed this impression, and these topics are no longer included.

As mentioned in section 3, there were also topics (such as planar integrals) that we considered core material, but which were not requested by any representatives. In this way, we found that the meetings helped make us conscious of what we thought we could not change. (An example of perceived or *internal constraints* (Artigue, 1994, p.32).) In the concrete case of planar integrals, we decided to keep the topic (but with less time spent on it), since we find that it adds to the understanding of functions of two variables and is useful in training integration skills.

Several other topics were up- or downgraded based on the responses from other subjects and our own experiences with the material. Also, some requested topics were left out because we considered them to be too advanced for the course (e.g. systems of differential equations, which are covered in another mathematics course at RVAU).

So far our discussion of topics has mainly dealt with selection details. However, the biggest change is undoubtedly that we now (in course material and lectures) use examples from applications as a starting point for the presentation of the mathematical theory; the main idea being to move "from concrete to general". In this way the examples serve two purposes:

- as motivation and to illustrate how a certain mathematical notion may turn up in other subjects. For example, models for soil temperature as a function of depth and time of the year were used to introduce functions of two variables.
- as applications of the mathematical theory. For example, as an application of investigation of functions, we maximised the profit in various models for harvest yield regarded as a function of the amount of fertilisers used.

The examples vary considerably when it comes to size and complexity. Moreover, many of the examples have been converted into exercises in order to allow the students to solve mathematical problems in applied contexts. The way the course is based on examples is one of the most visible changes from the students' point of view and they strongly appreciate it. Although it is hard to judge, our impression is that it has helped to improve the learning of the students.

Competencies and examples from applications

Clearly all of the eight competencies from the KOM report are relevant in varying degrees, but their relative importance depends on the context and aims of the course and should therefore affect the planning of the course. Partly based on the responses from the other subjects, we found that two of the eight competencies, namely the problem solving and the modelling competence (both from the group *being able to ask and answer in, with and about mathematics*) were more important in our course than the others. As another example of the relative importance of the competencies, we considered the reasoning competence (understanding mathematical proofs, etc.) to be the least important.

In the material for the course we gave brief descriptions of problem solving and modelling, and explained to the students that these competencies would be kind of "vertical themes" running through the course. We will now briefly describe these competencies and how we have attempted to train them.

The problem solving competence

This competence consists of being able to

- detect, formulate and specify different types of mathematical problems (pure and applied)
- solve such problems

In the list in section 3 of the abilities requested by the representatives, “handling equations” clearly falls under the problem solving competence: given an equation (typically from applications), the students should be able to understand what they must use it for and how. Moreover, problem solving in applied contexts should help to reduce the students’ fear of mathematics.

In order to help the students to think in a more organised way about problem solving, we often used the “What do I KNOW” – “What do I WANT” approach (Mason et al., 1985) in the material for the course. This approach was pointed out to the students in examples, and in exercises they were asked to clarify what they knew and what they wanted to know before attempting the exercises.

In many of the examples from applications, it is actually problem solving rather than modelling skills that are important, and we could therefore use these examples to train the problem solving competence. This was mainly the case when the students were presented with a mathematical description of a situation and from this had to draw various solutions.

The modelling competence
This competence consists of being able to

- structure the situation to be modelled
- perform a “mathematisation”, i.e. translate the situation into mathematics, resulting in a mathematical model
- analyse the model (here there is often a big overlap with problem solving)
- interpret the results and validate the model

The importance of this competence is rather evident from the context of the course and the needs of the other subjects as described in section 3.

Naturally, many of the examples from applications lend themselves to modelling. In our experience, it requires a lot of resources to teach the students active modelling, where the structuring and mathematisation of the situation play major parts. Because of this and the responses from the representatives (see section 3), we have chosen to play down this aspect. Instead we present the students with a fair number of models and let them work with similar models in exercises.

Training of competencies
In the first run of the course, we repeatedly emphasised the steps involved in modelling and problem solving during our presentation of examples in lectures, but did not explicitly train the competencies through exercises. The result was that although the students could see the two competencies as useful themes in the course, the competencies had actually only been trained to a limited extent. With hindsight our approach may seem naïve, but we were only beginning to find out how to approach the task. For the second run of the course, we constructed a number of exercises (often based on examples from applications) that more explicitly trained the competencies. It is our impression that this was a major improvement, which helped to bring competencies and subject matter together for the students. However, we are also convinced that this aspect of the course can be further improved, for instance through revised versions of the “mini-projects” (see below).

It is stressed throughout the KOM report that competencies should be regarded as a complement to and not a replacement of subject matter. In other words, it is the combination of subject matter and competencies that may lead to improved curricula, teaching and learning. Our experience with the interaction between competencies and subject matter is therefore not surprising, but highlights an important challenge associated with the introduction of competencies.

A more direct attempt to implement the interaction between competencies and subject matter has been made by Grønbæk et al. (2004), who introduced the notion of *specific competency goals* to describe what students should be able to do within a certain topic.

Practical organisation of the course

The course is divided into five modules according to topics ("Functions of one variable", "Matrices", "Statistics and probabilities", "Functions of two variables" and "Differential equations"), and the students find that this division helps them to get an overview over the course.

Ideally, we believe that the training and evaluation of competencies should partly be done by means of projects, but unfortunately we do not have the resources for this. The solution we have chosen is to end each module with a written "mini-project" inspired by an example from applications. Strictly speaking this is merely a large exercise that does not involve project work, but it sums up the module and gives the students a sense of progression. The mini-projects are handed in and marked, but do not directly give credit in the course. In the four hour written exam at the end of the course, about 35% is taken up by a question, which in style and content resembles one of the mini-projects. Generally the students do well in this question, which we take as indication that they have worked well on the mini-projects during the course.

5. References

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